

THE QUALITATIVE ANALYZE OF THE CARTOGRAPHICAL PROJECTIONS USED IN ROMANIA

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***Abstract:** The basic documents made after the geodesic and topographical measurements for accomplishing different projects are the topographical plans and maps. The accomplishment of these documents must represent very good of the terrestrial area with representative elements. Using the cartographic projections that must be made depending on the quality of the representation so that the plans and maps, which are obtained, should be made with high precision and high quality makes these documents. The elements that are deformed are as follows: angles, distances and areas.*

Keywords: Gauss Kruger, UTM, Stereographic, projection

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1. INTRODUCTION

To choose a projection system is made depending on the size and shape of the area that should be represented. So, for the mid territories, whose shape is almost round we can use the stereographic projections, and for the big ones, as continents, we can use the cylindrical and transversal projection Gauss-Kruger. The military and engineering needs are satisfied by the conform projections, and for surveying needs we can use the equivalent projections, and for the maritime and air navigation it is used the Mercator projection.

In Romania there were used along the time the following projection systems: Conic projection Bonne, on the Clarke ellipsoid, between 1916-1930, Stereographic projection on the Hayford ellipsoid, between 1930-1951, **Gauss-Kruger** projection, on the Krasowsky ellipsoid, between 1951-1970, **Stereographic projection 1970**, on the Krasowsky ellipsoid, starting with 1971. For the military applications in Romania it is used the **UTM projection (Universal Transverse Mercator)**.

2. CYLINDRICAL TRANSVERSAL GAUSS-KRUGER PROJECTION

2.1. The geometric elements of this representation

This projection is characterized that a certain area of the terrestrial surface is represented on a cylinder tangent or transversal to the surface of reference which is considered a sphere (Fig. 1).

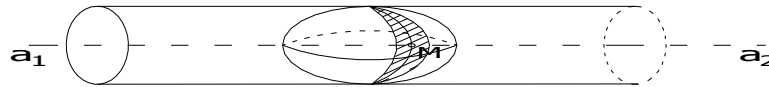


Figure 1 The projection cylinder

In a certain position of the reference surface inside the cylinder this is tangent after the median line. This line after it is developed into the plane of the cylinder remains unreformed. It is deformed any element of length situated on a side and the other side of the tangent meridian if the distance is big. By being imposed the limit of 1/2500, it results that the representing area is limited to 6° and it is named shaft of representation. By imaginary rotting of the reference surface inside the cylinder it can be obtained 60 shafts numbered from 1 to 60 over the meridian 180°. In these conditions our country belongs to the shafts 34 and 35 limited by the meridians 18° and 24° respective 24° and 30°. The reference surface is divided in bends having the width of 4° numbered for each hemisphere with letters from A to V. being situated at the latitude of 45° of the north hemisphere, our country belongs to the bend L, with small regions in bends K and M, limited by the parallels 44° and 48°.

By such deviation it results trapezes having the sizes of 4°X4° noted for our country with L-34 and L-35. The reference system which establishes the positions in plane of the points represented on the developed surface of the cylinder is xOy. (Fig. 2)

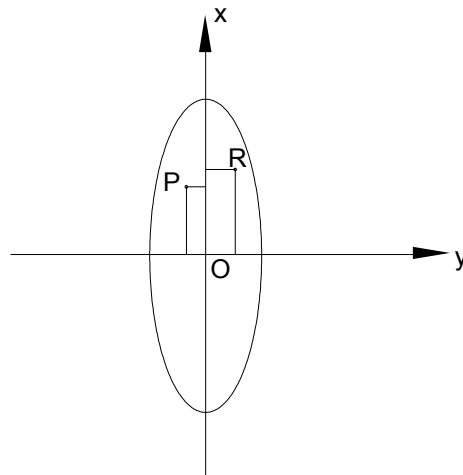


Figure 2 The rectangular reference system in Gauss – Kruger projection

The x ax is oriented on the direction of the axial meridian of the shaft, and the y ax is on the direction of the equator. In order to have the points inside the shaft to have the coordinates positive, the x ax is moved to West with 500 km so that the origin has the coordinates 0 km and 500 km.

The writing of the coordinates is made as follows:

For R point:

$$\begin{aligned} x_1 &= x_1^C \\ y_1 &= n(500 \text{ Km} + y_1^C) \end{aligned} \quad (1)$$

For P point:

$$\begin{aligned} x_2 &= x_2^C \\ y_2 &= n(500 \text{ Km} - y_2^C) \end{aligned} \quad (2)$$

where:

- x_1^C, x_2^C - is the calculated value and represents the distance between the points R and P to the equator
- y_1^C, y_2^C - is the calculated value and represents the distance between the points R and P and the axial meridian
- n – is the number of the shaft to the Greenwich (4 or 5 for our country); n does not multiply the parenthesis

2.2. Distance Deformations

In order to establish the deformations in Gauss – Kruger deformations it is used:

The module of linear deformation:

$$\mu = 1 + \frac{y^2}{2R_m^2} = \frac{dS}{ds} \quad (3)$$

where: dS is the linear element in plan and ds is the linear element on the ellipsoid
 It results :

Unitary deformation $\Delta = \frac{dS - ds}{ds} = \frac{y^2}{2R^2} \quad (4)$

If at medium latitude of our country the shaft of 1^o has the width of 80 km, in the limitation area of the shaft that means at 3^o over the axial meridian the deformation on km may reach 0.70m/km. for the representations at great scales the deformation is so great that we can use the shafts of 3^o when at the limit the deformation on km is only of 0.18 m/km.

We can notice from the following diagram (Fig. 3) that into the Gauss projection the linear relative deformations are positive and proportional with the distance over the axial meridian.

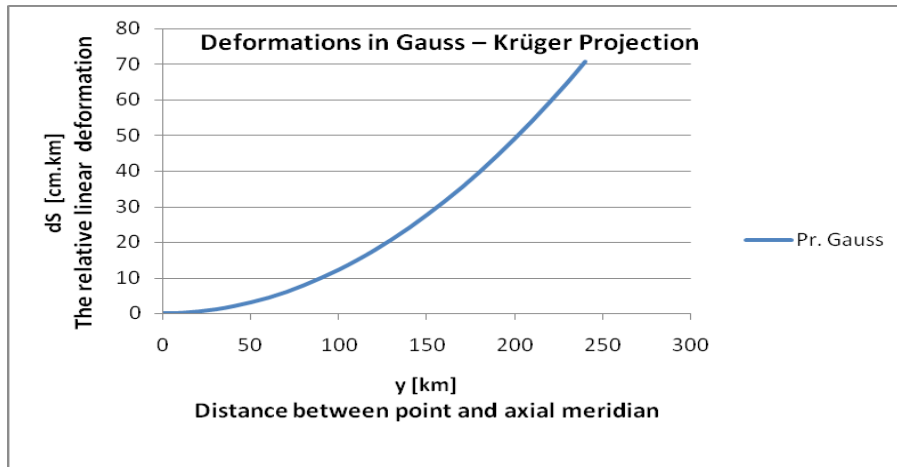


Figure 3 Diagram of deformations in Gauss-Kruger projection

3. STEREOGRAPHIC PROJECTION 1970

3.1. Geometric elements of this representation

The stereographic representation (projection) is characterized that a certain part of terrestrial surface is represented on the surface of a plan tangent or secant on the reference surface (fig. 4).

Geometric elements of the representation are:

- H – projection plan tangent or secant to the reference surface;
- C – projection center;
- O_1 – point of view where start the projection rays situated on the reference surface opposed to C point;
- P – point which is represented;
- P' – projection of P point on H plan.
- x ax on the direction of the C point meridian;
- y ax on the direction of the C point parallel;

In order that all coordinates should be positive the origin of axes that are moved and becomes O (500 Km ; 500 Km).

The geographic coordinates of C point are:

$$\Phi_0 = 46^0; \quad \lambda_0 = 25^0$$

The C point is situated near Făgăraș town.

For a certain point P situated on the reference surface having the plan coordinates x and y in *stereographic projection* with tangent plan are as follows:

$$\begin{aligned} x &= a_{10} \Delta \varphi + a_{20} \Delta \varphi^2 + a_0 2\beta + \dots \\ y &= b_{01} l + b_{11} \Delta \varphi l + b_{21} \Delta \varphi^2 l + \dots; \end{aligned} \quad (5)$$

where:

$a_{10}, a_{20}, \dots, b_{01}, b_{11}, \dots$ are coefficients calculated for the latitude of C point;

l – are differences between the latitude and longitude of P point and C point.

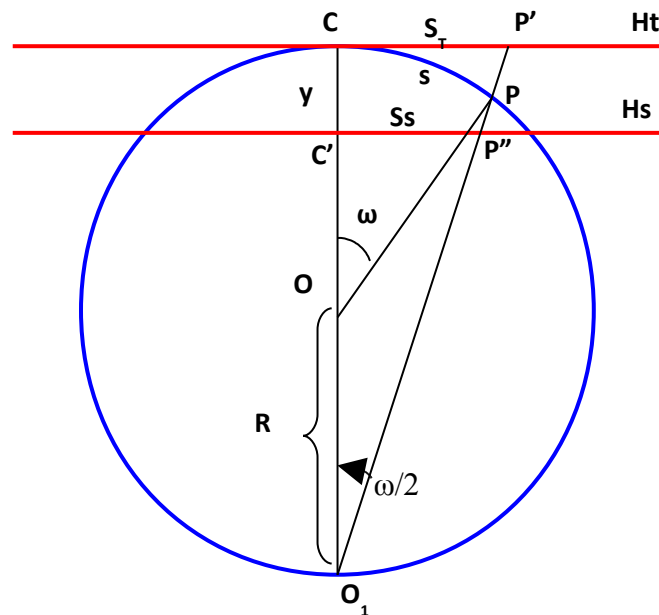


Figure 4 Stereographic projection with tangent and secant plan

3.2. Distances deformations

From fig. 4 it can be written as follows:

$$(6) \quad S_T = 2 R \operatorname{tg} \frac{\omega}{2}; \quad \text{is developed in Taylor series}$$

By differencing it can result:

$$dS = \left(1 + \frac{s^2}{4R^2} \right) ds; \quad (7)$$

Unitary deformation

$$\Delta = \frac{dS - ds}{ds} = \frac{s^2}{4R^2}. \quad (8)$$

By comparing the mentioned relations it results that the unitary deformations in stereographic projection with tangent plan has values smaller than the deformations from cylindrical projection. So it is qualitatively superior to this. For the stereographic projection with secant plan we have the following relations:

$$S_s = R\omega + \frac{(R\omega)^3}{12R^2} - \frac{R\omega}{2R} y = s + \frac{s^3}{12R^2} - \frac{s}{2R} y. \quad (9)$$

Unitary deformation

$$\Delta = \frac{dS - ds}{ds} = \frac{s^2}{4R^2} - \frac{y}{2R}. \quad (10)$$

By comparing the mentioned relations we can notice that the unitary deformations in stereographic projection with secant plan are smaller with the quantity $\frac{y}{2R}$ over the ones from the stereographic projection with tangent plan.

The value of this deformation is established as follows:

In stereographic projection with tangent plan the maxim unitary deformations result for $s = 275$ Km, and the minim ones for $s = 0$.

So: for $s = 0$; $\Delta = 0$ and for $s_{\max} = 275$ Km; $\Delta_{\max} = \frac{1}{2000}$.

There are reduced with $\frac{\Delta_{\max}}{2}$ the deformations from above and result: $\Delta_{\max} = \frac{1}{4000}$

So: $\frac{y}{2R} = \frac{1}{4000}$ and $y = 3.189$ Km; (11)

There will be a circle of null deformations with the following ray:

$$\frac{s^2}{4R^2} - \frac{y}{2R} = 0; \quad s = \sqrt{2Ry} = 201,718 \text{ Km}; \quad (12)$$

3.3. Transformation of the coordinates from tangent plan into secant plan

If for certain point situated on the reference surface, the coordinates in the two systems of projections we can write with x_t, y_t and x_s, y_s as follows:

$$\begin{aligned} X_S &= X_T - \frac{X_T}{4000}; Y_S = Y_T - \frac{Y_T}{4000} \\ X_S &= \left(1 - \frac{1}{4000}\right)X_T = KX_T; Y_S = \left(1 - \frac{1}{4000}\right)Y_T = KY_T \\ K &= \left(1 - \frac{1}{4000}\right) = 0.999750 \end{aligned} \quad (13)$$

From the above mentioned equations it can also be made the inverse transformation as follows:

$$X_T = \frac{1}{K} X_S; Y_T = \frac{1}{K} Y_S; \frac{1}{K} = 1.000250063 \quad (14)$$

The linear local deformations depending on the distance over the central point of the Stereo 1970 projection are shown in the table 1 ($R_0=66378956,681$ m)

Table 1 The distance over the central point of the Stereo 1970 projection

S [km]	Region deformation [cm/km]	Relative deformation
0	-25	1/4000
1	-25	1/4000
10	-24.94	1/4010
20	-24.75	1/4040
30	-24.45	1/4090
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200	-0.42	1/38000
201.718	0	-
210	2.09	1/47800
220	4.74	1/21000
230	7.5	1/13000
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430	88.6	1/1100

The curve of regional (local) deformations on secant plan is shown in the following diagram (Fig. 5)

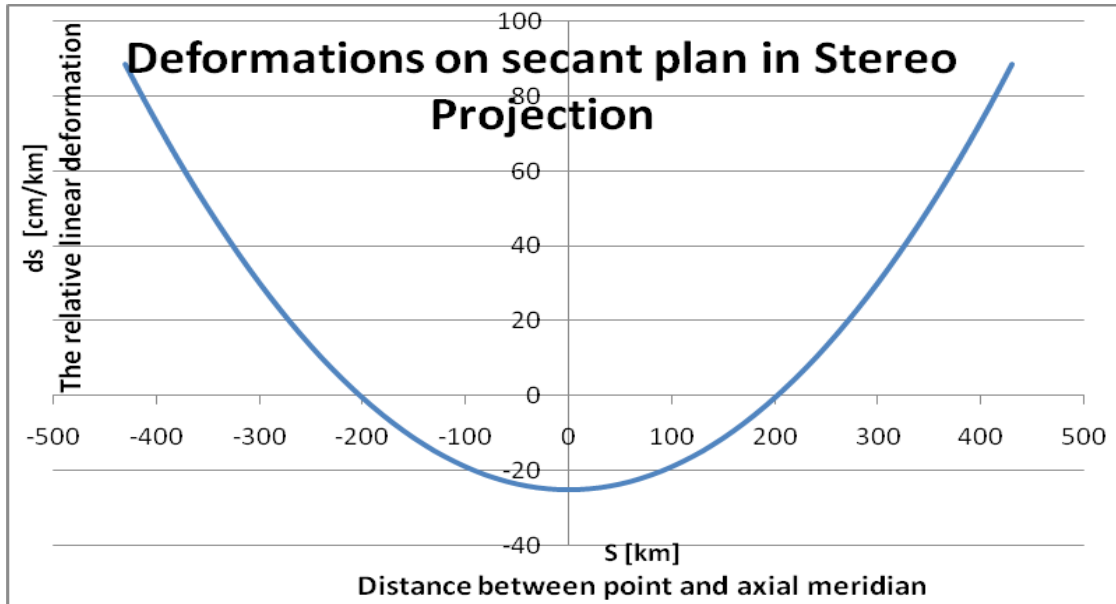


Figure 5 Diagram of deformations in Stereo70 projection

4. UTM PROJECTION (UNIVERSAL TRANSVERSAL MERCATOR)

This projection is a variant of the *Gauss – Krüger* projection, used in United States of America and in other countries, being important also for Romania due to our integration in new political and military structure.

The cartographical representation is made on fuses of 6° longitude, into the interval delimited by the parallels of 80° latitude south and 84° latitude north. The reference ellipsoid is the international ellipsoid called *WGS – 84*, for which:

Great semi ax: $a = 6378137,000 \text{ m}$

Geometrical flattening: $f = 1 / 298,257223563$

The projection cylinder (Fig. 4) is modified by reducing its dimensions and bringing it into secant with the ellipsoid along of 2 lines parallel with the central meridian. This means that in an area of 6 degrees there are 2 line of secant situated of almost 180 000 m E and V from the axial meridian. In order avoid to use the negative coordinates, to the central meridian it is attributed a false value of the east of 500 000 m E, this leading to the values of 320 000m E and respective 680 000 m V for the two secant lines.

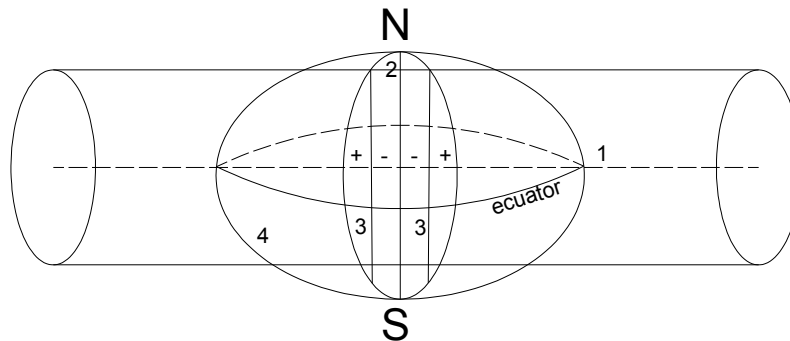


Fig. 6 Projection cylinder

- 1 – ax of the cylinder situated in equatorial plan;
- 2 – axial meridian;
- 3 – secant meridians;
- 4 – margin meridian of the fuse of 6 ;
- flexion ray of the meridian ellipse of φ latitude

- meridian arc β between 2 parallels φ_1 and φ_2

Adopting the representing system on fuses of 6° longitude, the plan representation is almost fiddled. Using the Mercator projection which is conform projection because the angles and the module u of linear deformation are not deformed very much it leads to a precise representation of the earth. Its disadvantages are that the calculation is very difficult but due to the modern technology of calculation this is not observing.

4.1. The deformations in UTM projection

The relative linear deformation has the following formulas:

$$D_{UTM} = k(D_{Gauss} + 1) - 1 = k(L^2 / 2R^2 + L^4 / 24R^4 + 1) - 1 \text{ [cm/km]} \quad (15)$$

where:

- D_{UTM} relative linear deformation in UTM projection;
- D_{Gauss} relative linear deformation in Gauss projection;
- R is the average ray of flexion into the considered point;
- $y=(y-y_0)$ is distance between the point and axial meridian;
- k is the constant report between the distances from the UTM projection plan and the ones from the Gauss projection plan.

By using this formula for the relative linear deformation in UTM projection there are obtained values which are directly proportional with the distance against the axial meridian and they increase starting with the negative value -40 cm/km (Fig. 7)

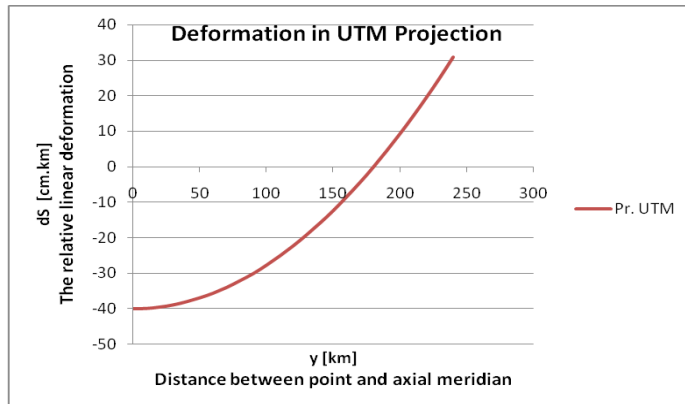


Figure 7 The diagram of relative linear deformation in UTM projection

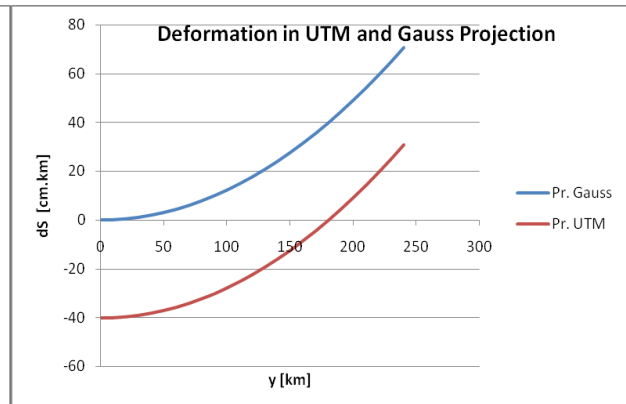


Figure 8 Comparative Diagrams of the relative linear deformation in UTM and Gauss projection

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